AP® Calculus BC

Course Overview

My main objective in teaching AP® Calculus BC is to provide students with an opportunity to explore the higher levels of mathematics. Through this exploration and interaction with mathematics I hope to enable students to appreciate the higher intricacies of problems, and develop a solid foundation in the Calculus BC topic outline as it appears in the *AP® Calculus Course Description*, which they can take with them into their higher level classes. I expect a lot of my students, whether that is in class in discussion and group work time, or at home writing up assignments and AP sample problems.

In order to best teach all students, I strive to present all topics in many different ways. Among these are graphical, numerical, analytical and verbal approaches to almost all problems. I use virtual TI programs for both the TI-83 plus and TI-89 titanium projected onto the board regularly to give students an idea for the big picture of a problem, often showing students how to use the table feature, or math menu (zeros, derivative at a point, or numerical integral) in graphical mode, to get a more numerical approach to problems. All students are required to have a graphing calculator, with about half of the class using a TI-83+ or TI-84+ and half purchasing a TI-89. If students are unable to purchase a calculator, we have a set of TI-83+ calculators which I can check out to students for the year. Finally, I use Geometer Sketchpad, and *Calculus in Motion* by Audrey Weeks to help students visualize the different topics I am presenting, specifically for different theorems and Solids by revolution.

Assessment

Students are assessed in my class daily, weekly, and once a unit. Each day, students have an assignment addressing the topics we covered in class. Each week students receive a "packet of the week" which has problems from released AP exams, both multiple choice and free response designed to target the subject from the week before. On both of these assignments (homework and Packets of the Week) students are encouraged to work together to complete the problems, but are required to write up their own solutions to problems. Once or twice each unit we will have a half period test on material covered up to the day before. On these assessments students are allowed to use a calculator about half of the time. At the end of each unit students are given an exam covering all material in the unit. These exams are generally written in two parts, one calculator active, and one without calculators. Problems are generally open ended, and students are required to show work to get full credit. Often there is at least one released Free Response question from previous AP exams, which is scored in the same manner as they will be on the AP exam.

Course Format

Students enrolled in AP® Calculus BC have successfully completed AP® Calculus AB as a junior. We generally have a handful of students who are seeking to further their mathematical knowledge and are looking for a challenge for their senior year. Students enroll in second year calculus as an "Independent Study." Students are scheduled into my

class during the same period as Calculus AB, and are expected to participate in class regularly. Calculus BC students serve as tutors for the AB students, working with students when they are stuck on a particular problem or concept. This enables students to stay current on their skills from the year before, and provides new explanations for students enrolled in AB. Calculus BC students are given separate assignments, focusing on the different topics covered in the BC curriculum. Because of the different nature of this course, I have included my AB course outline, followed by the outline I use with my BC students. Students in BC typically will complete the chapter reviews for each unit in AB, as well as their own materials, further keeping them up to speed on previous skills.

Textbooks

Students in BC calculus use primarily one textbook:

Larson, Ron, Hostetler, Robert P., and Edwards, Bruce H. *Calculus of a single variable*. 8th edition. Boston, Houghton Mifflin Company. 2006.

Students in AB calculus use two textbooks, the Larson book as well as:

Foerster, Paul A., *Calculus Concepts and Applications*. 1st edition. Emeryville, CA: Key Curriculum Press. 1998.

AB Course Outline:

Foer stands for our Foerster book, and LHE stands for our (4th edition) Larson, Hostetler, Edwards book.

Unit 1 – Limits

(13 days)

- Foer 1-1 – average vs. instantaneous rate – estimating limits
- LHE 2.1 – tangent line problem/intro to limits – table, graph and algebra
- LHE 2.2 – properties of limits
- LHE 2.3 – techniques for evaluating limits
- LHE 2.4 – continuity and intermediate value theorem
- Foer 2-4 – continuity of piecewise functions
- LHE 2.5 – infinite limits and vertical asymptotes
- LHE 4.5 – limits at infinity and horizontal asymptotes

Unit 2 – Concept of the Derivative

(14 days)

- Foer 1-2 – rate of change by equation, graph, and table
- Foer 3-1 – graphical interpretation of derivative
- Foer 3-2 – difference quotient, derivative at c
- Foer 3-4 – derivative at x, properties of derivatives
- Foer 3-5 – displacement, velocity and acceleration
- Foer 3-6 – intro to sine, cosine, composite functions
- Foer 3-7 – chain rule
- Foer 3-8 – writing sinusoidal equations

Unit 3 – Derivative Formulas

(15 days)

Foer 4-2 – product rule
Foer 4-3 – quotient rule
Foer 4-4 – trigonometric functions
Foer 4-5 – inverse functions, inverse trigonometric functions
Foer 4-6 – differectiability and continuity, piecewise function
Foer 4-8 – implicit differentiation
LHE 3.7 – related rates
Foer 10-4 – related rates

Unit 4 – Graphical Analysis

(23 days)

LHE 4.1 – extrema, extreme value theorem
LHE 4.2 – Rolle's Theorem, Mean Value Theorem
Other sources (Ostebee and Zorn) – plotting derivatives, geometry of derivatives, geometry of second derivatives
LHE 4.3 – increasing/decreasing, first derivative test
LHE 4.4 – concavity, point of inflection, second derivative test
LHE 4.6 – curve sketching
LHE 4.7 – optimization
Foer 10-5 – minimal paths

Unit 5 – Integrals

(21 days)

Foer 1-3 – intro to definite integrals
Foer 5-1 – intro to definite integrals
Foer 3-9 – antiderivatives and indefinite integrals
Foer 5-2 – review of antiderivatives
Foer 5-3 – linear approximations and differentials
Foer 5-4 – antiderivatives and indefinite integrals, u-substitution
Foer 1-4 – trapezoid rule (including unequal intervals from outside source)
Foer 5-5 – Riemann sums and definite integrals
Foer 5-6 – Mean Value Theorem and Rolle's Theorem
Foer 5-7 – special Riemann sums
Foer 5-8 – Fundamental Theorem of Calculus
Foer 5-9 – properties of definite integrals
Other sources – Functions defined by integrals/FTC 2/Area Function

Unit 6 – Exponential and Logarithmic Equations

(17 days)

LHE 6.1 – natural logarithmic function and definition of e
LHE 6.1 – derivative of natural logarithm, logarithmic differentiation
LHE 6.2 – antiderivatives of reciprocal function and trigonometric functions
LHE 6.3 – inverse functions

LHE 6.4 – derivative of natural exponential function, antiderivatives of
LHE 6.5 – exponential functions with other bases

Unit 7 – Applications of Integrals
(15 days)

LHE 7.1 – area between two curves
LHE 7.2 – volume by discs, washers and known cross-sections
Foer 10-2 – distance vs. displacement, speed vs. velocity
Foer 10-3 – average value

Unit 8 – Differential Equations
(11 days)

Differential Equations and Slope Fields will be covered using materials from other books in my collection. Both are not covered particularly well in either of our textbooks.

Topics covered:
Solutions (general and specific) to separable differential equations
Slope Fields
Exponential Growth and Decay – specifically as it relates to modeling

Unit 9 – AP Exam Review
(22 days)

We spend one or two days on each of the previous units, with some sample questions given to students each day. For homework, students write 5 questions on notecards, with full solutions on the back. From these questions, we review as a class each topic. We play a "chess" game where pieces move according to specific functions across the board, and to take a person's piece you must answer a question correctly. This encourages students to dialog together, as they discuss why an answer is correct, including defending their position if their answer is different from the stated answer.

BC Course Outline:

Timeline is approximate, and includes review and testing; students are completing many of the AB assignments, and are teaching themselves each new topic. This encourages students in the BC program to work together and develops a mathematical maturity and independence before they go to college. I am there as a resource to them, just as any textbook or other outside source they might find.

Unit 1 – Planar Curves
(40 days)

LHE 7.4 – Arc Length and Surfaces of Revolution
LHE 10.2 – Plane Curves and parametric Equations
LHE 10.3 – Parametric Equations and Calculus
LHE 10.4 – Polar Coordinates and Polar Graphs
LHE 10.5 – Area and Arc Length in Polar Coordinates

LHE 11.1 – Vectors in the Plane
LHE 11.2 – Space Coordinates and Vectors in Space
LHE 12.1 – Vector-Valued Functions
LHE 12.2 – Differentiation and Integration of Vector-Valued Functions
LHE 12.3 – Velocity and Acceleration

Unit 2 – Techniques and uses of derivatives and antiderivatives
(30 days)

LHE 6.1 – Slope Fields and Euler's Method
LHE 6.3 − Separation of Variables and the Logistic Equation
LHE 8.2 − Integration by Parts
LHE 8.5 − Partial Fractions
LHE 8.7 − Indeterminate Forms and L'Hôpitals Rule
LHE 8.8 − Improper Integrals

Unit 3 – Polynomial Approximations and Series
(40 days)

LHE 9.1 – Sequences
LHE 9.2 – Series and Convergence
LHE 9.3 – The Integral Test and *p*-series
LHE 9.4 – Comparisons of Series
LHE 9.5 – Alternating Series
LHE 9.6 – The Ratio and Root Tests
LHE 9.7 – Taylor Polynomials and Approximations
LHE 9.8 – Power Series
LHE 9.9 – Representations of Functions by Power Series
LHE 9.10 – Taylor and Maclaurin Series

Unit 4 – Review

Review will be similar to AB review above, students will be given short assignments and asked to write practice problems, as well as take several full released exams.